Model Checking (SS 2025) Homework 7

Deadline: May 13, 2025, 9:00 am
Submit your solution through TeachCenter

Task 1. [40 points] Intersection of Büchi automata.
Consider the algorithm to construct the intersection of two Büchi automata that we discussed in the lecture and that is discussed in the book at page 89.

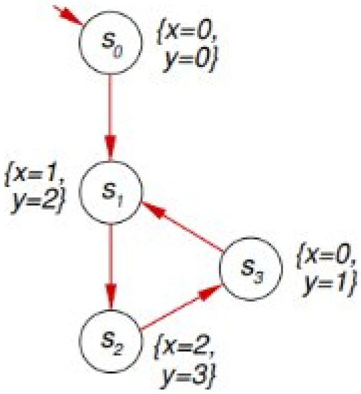
The algorithm uses a variable $x \in \{0,1,2\}$ to create 3 copies of the state space to track if accepting states of both automata are visited infinitely often.

Your Task: Change the algorithm such that $x \in \{0,1\}$. In particular, think how you need to define the **transition relation** and the **set of accepting states** of the product automaton, if you now only have two copies of the state space available.

Task 2. Consider the following LTL properties φ_1 and φ_2 and the Kripke structure M

$$\varphi_1 = F(x = 1 \wedge y = 3)$$

$$\varphi_2 = F(y = 2 \wedge X\, x = 2)$$



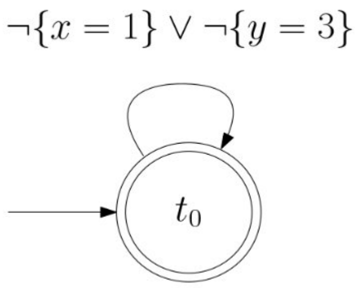
Task 2a. [30 Points] Check whether it holds that $M \models \varphi_1$.
Task 2b. [30 Points] Check whether it holds that $M \models \varphi_2$.

Use the algorithm that we discussed in the lecture. You also find the algorithm on page 98 (chapter 7) of the Model Checking book. Give (a few) details of your computation.

1. Construct $\neg\varphi$
2. Construct a Büchi automaton* $\mathscr{S}_{\neg\varphi}$
3. Translate M to an automaton $\mathscr{A}$.
4. Construct the automaton $\mathscr{B}$ with $\mathscr{L}(\mathscr{B}) = \mathscr{L}(\mathscr{A}) \cap \mathscr{L}(\mathscr{S}_{\neg\varphi})$
5. If $\mathscr{L}(\mathscr{B}) = \varnothing \Rightarrow \mathscr{A}$ satisfies φ
6. Otherwise, a word $v{\cdot}w^{\omega} \in \mathscr{L}(\mathscr{B})$ is a counterexample

*You can skip step 2 and use the Büchi automata given below for $\neg\boldsymbol{\varphi}_1$ and $\neg\boldsymbol{\varphi}_2$.

Automaton for $\neg\boldsymbol{\varphi}_1$:



Automaton for $\neg\boldsymbol{\varphi}_2$: